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MEASUREMENT ASSURANCE

IN

YOUR OWN LABORATORY

by

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ABSTRACT

Measurement assurance is the application of broad quality control principles to measurements and calibrations. This paper demonstrates how quality control methods can be used to control the accuracies of measurements and calibrations.

CONTROLLING A PROCESS

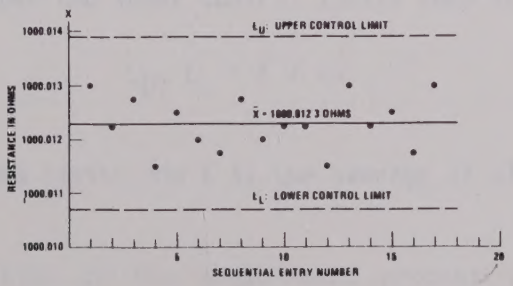
In order to manufacture a number of like products efficiently, a manufacturing process must be established that consistently turns out the product with the desired characteristics at a predetermined cost. Once the process is established, one task of quality control is to keep it operating at the intended levels of product quality and production cost. Inadvertent changes in the process must not be permitted to occur; and if such changes do occur, steps must be taken to prevent their recurrence. To this end, numerous production control techniques have been invented which can also be used to control measurement and calibration processes. Ever since Eisenhart (Ref. 1) has shown that a measurement process, to yield measurements with known limits of variability, must behave like a production process which is in a state of statistical control, it has become very useful to consider a measurement process like a production process and apply to the measurement process controls similar to those first used to control manufacturing processes.

THE CONTROL CHART

One such control instrument is the control chart. In this paper, we shall examine how control charts can be used to control a measurement process. The word "measurement" is used here in a broad sense so as to include calibrations. In fact, the methods described here have primarily been used to control calibration processes.

The most widespread use of control charts in controlling measurement processes is in connection with the use of a check standard. To control a measurement process, a check standard is measured from time to time by the process to be controlled and its measured value entered on the chart. If more than one observation is normally made to determine a measured value, the dispersion of the observations is also determined numerically and can be entered on an accompanying chart to furnish an additonal control element.

As an example, refer to Figure 1, the control chart of a 1-ohm standard resistor used as a check standard for resistance measurements made by the same process. Figure 1 illustrates the simplest form of a control chart; other forms of control charts, yielding more or different information, will be introduced later.



Control Chart of a One-Ohm Resistor Used as a Check Standard

Figure 1

In Figure 1, each point, X_i (i = 1, 2, 3...), represents one resistance measurement which, in this case, consists of one observation only. The center line is the average of the measurements and is indicative of the value towards which our measurement process of the resistor tends, in our case 1.000 012 3 ohms. (This value may be used as our best estimate of the true value of the resistor or it may be used to compare our measurement process with that of a higher echelon laboratory. If, for instance, the charted resistor has been certified by the National Bureau of Standards to a value of 1.000 009 5 ohms with an uncertainty of 2 microohms, the difference points to a possible systematic error in our measurement process or a changed value of the resistor or both.)

The dashed lines in Figure 1 are upper and lower control limits. They are usually drawn at three standard deviations (3-sigma) so that 99.73 percent of all measurements would theoretically fall within those limits, provided the measured values plotted come from a normally distributed population. Thus, a point falling outside these limits is always highly suspect of coming from a different population, suggesting a change in the measurement process or in the resistor, or revealing some operator error. Such an occurrence should be investigated immediately to determine the error cause and to correct the measurement value.

The equation for upper and lower control limits then is

$$L_U, L_L = \bar{X} \pm Z\sigma \tag{1}$$

where Z=3 for three-sigma limits and $\bar{X}$ is the average of all plotted points X_i.

The standard deviation of the measurement process is either known (σ) from past experience or it is estimated (s) from the plotted points using equation 2.

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n}X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n}X_i\right)^2}{n-1}} \tag{2}$$

The first expression for s represents the definition of the term and is widely used to calculate s. The right hand expression for s is more easy to use in manual and calculator computations and gives certain advantages in computer calculations. Since s is a function of a small difference between two large numbers, different results may be obtained at times when calculating s with one or the other of the two expressions of equation 2 because of the effect of rounding. To avoid such differences, it is advisable to use as each value of X_i only the variable part of X_i in the calulation of s. For example, if $X_1 = 1004$, $X_2 = 1006$, $X_3 = 995$, etc., use as X_i only 4, 6, -5, etc.

The term $\left(\sum_{i=1}^{n} X_i\right)^2$ stands here for the sum of all n measured values of X, squared after summation:

$$\left(\sum_{i=1}^{n} X_i\right)^2 = (X_1 + X_2 + \ldots X_n)^2$$

And $\sum_{i=1}^{n} X_i^2$ stands for the sum of all squared values of X,

$$\sum_{i=1}^{n} X_i^2 = X_1^2 + X_2^2 + \ldots X_n^2.$$

If equation 2 is used to estimate the standard deviation of the n plotted points X_i, s replaces sigma (σ) in equation 1. Simultaneously, the factor Z must be replaced by Student's t which makes allowance for the limited number of data from which the estimate of sigma is derived, because Z applies strictly only when the standard deviation is computed from a very large number of data points. Hence, for a limited number of points, equation 1 becomes

$$L_U , L_L = \bar{X} \pm s\, t_{P,\, n-1} \qquad (3)$$

where $t_{P,\, n-1}$ is the two-sided P-probability value for Student's t for n-1 degrees of freedom. For upper and lower control limits, $t_{0.9973}$ (corresponding to the normal deviate Z = 3.0) or $t_{0.995}$ are recommended. When the

number n of all points X_i is small, the equivalent of two-sigma control limits is often used; in such cases, $t_{0.9544}$ (corresponding to the normal deviate Z = 2.0) or $t_{0.95}$ is applied at the slightly increased risk of rejecting erroneously a perfectly valid measurement value falling outside the control limits. Many textbooks contain tables of Student's t factors for various values of P and various degrees of freedom (in this case, n-1). However, we owe to Brian Joiner (Ref. 3) the calculation of t factors corresponding to whole numbers of Z. Table I lists some values of t.

The distance of the control limits from the center line ($\bar{X}$) either calculated from multiplying the known standard deviation by three or from equation 3, represents also the limits of the random error of the charted measuring process. To this random error, we simply need to add the known or estimated limits of systematic error to arrive at the total uncertainty limits of the measuring process.

When the standard deviation of the measuring process is known and some points repeatedly fall outside the control limits, the process is out of control and should not be used for measurements before the causes for the out-of-control condition have been removed. Such a change in the measuring process, however, may also require a re-evaluation of the random error of the measuring process.

An important condition for the use of control charts is that the measurement process remains the same throughout. Different measurement processes require separate control charts. Thus, a control chart monitors the entire measurement process, in our example the resistor and the system measuring it. In fact, a control chart for a standard resistor will show an out-of-control situation developing when for instance the oil of the bath in which it is kept loses its insulating qualities, even though the resistor and the other parts of the measuring system have remained stable.

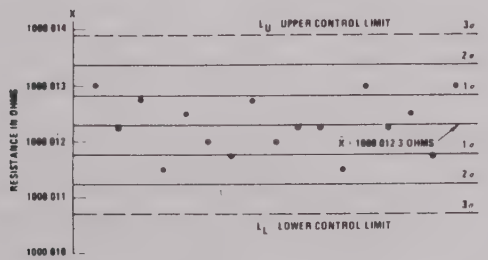
The control chart can yield considerably more information, however. In addition to identifying a measurement with a likely excessive error, it can

signal promptly a slightly changed measurement process, even though individual measurements may still lie within the expected uncertainty band. For this reason, frequently one- and two-sigma lines are drawn on the chart between the control limits as shown in Figure 2. With known standard deviation (sigma), the band between a control limit and the center line is simply divided into three equal parts. When the standard deviation is estimated as in equation 2, one- and two-sigma equivalent lines are drawn using equation 3 and $t_{0.6826}$ and $t_{0.9544}$ respectively or, if these t-factors are unavailable, $t_{0.70}$ and $t_{0.95}$.

With one- and two-sigma lines we can spot a bias which did not exist before, a developing out-of-control condition, or a changed systematic error, when

a. two out of three successive points fall beyond the two-sigma line,
b. four out of five successive points fall beyond the one-sigma line, or
c. eight successive points fall on one side of the center line.

The measurement process should be investigated for a probable cause changing the results of the measurements when one of the above conditions has occurred, because conditions a, b, and c have only a 0.16%, 0.32%, or 0.4% chance of occurring when the measurements are normally distributed.



Control Chart with One-, Two-, and Three-Sigma Lines
Figure 2

TREND CHARTS - CONTROL CHARTS FOR REFERENCE STANDARDS

STABLE STANDARDS

Control charts for reference standards are also called Trend Charts. They are an indispensable tool to

1. assign a best value to a measurement standard and thus prevent a laboratory from making the common mistake of always assigning to the standard the latest value reported by the higher echelon calibration laboratory (e.g. NBS);

2. estimate the total uncertainty of the standard (which is in most cases larger than the calibration uncertainty reported by the higher echelon laboratory) and thus avoid another common oversight;

3. indicate the relative stability or drift (trend) of the standard or, when only a few points are available, give the metrologist some indication of what behavior he might expect of it in the future and, therefore, help to

4. determine the date when the reference standard will have to be recalibrated or recertified.

They thus furnish the key to a wealth of information embodied in seasoned reference standards that would otherwise be disregarded and remained locked into them were it not for Trend Charts.

In fact, a laboratory which assigns to its standards the value obtained from the latest calibration of that standard introduces in most cases an out-of-control condition into their measuring system while considerably underestimating the uncertainty of their standard.

As an example, consider the Trend Chart of a one-ohm standard resistor, Figure 3. All values were reported to us by the National Bureau of Standards (NBS) as the mean of two observations each with a known standard deviation of random error of approximately 0.2 microohms. The standard deviation of each mean of two measurements, therefore, is calculated to be

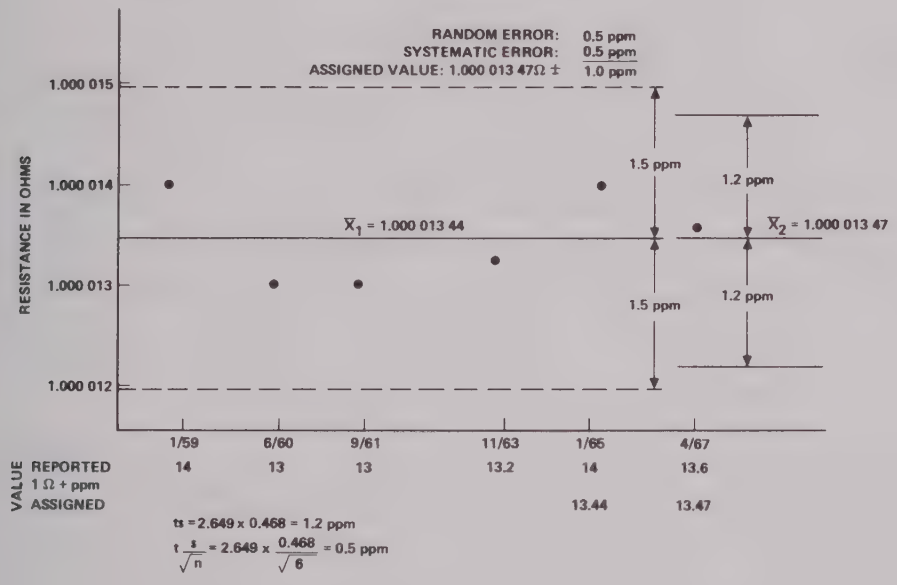
$$\frac{\sigma}{\sqrt{n}} = \frac{0.2}{\sqrt{2}} = 0.14 \text{ microohms.}$$

Control limits drawn at three standard deviations of NBS's random error would, therefore, fall about 0.4 microohms from the center of trend line, excluding most of the points. Such a control chart would show a process that is out-of-control -- which is certainly not the case. But we know that the standard probably undergoes small changes over long periods and may be influenced by additional short-term fluctuations. There is, of course, also the non-negligible probability that NBS's systematic error changes somewhat over longer periods. In sum, the uncertainty of the value of the standard is generally larger than the uncertainty of the value reported by the calibrating laboratory. All the sources contributing to the apparent fluctuations of the standard's value over longer periods whose causes are not identifiable, including the random error of NBS's measurements, can be considered as belonging to one cause system. We can, therefore, estimate the magnitude of the combined effects causing the random fluctuations between calibrations from equation 2.

The estimate of the standard deviation of the first five points ($n_1 = 5$) calculated from equation 2 is $s_1 = 0.518$ microohms. Since only five points were available, two-sigma limits were drawn at 1.5 microohms from the center line. These limits should be considered as limiting the band within which all expected future values should fall if the standard remains stable and the sources causing the variability do not change.

The best known value of the resistor, in the absence of any discernible drift, is the average of the recorded values, provided we have no reason to discard any of them. Beginning January 1965, the value to be assigned to the standard plotted in Figure 3 was 1.000 013 44 ohms. In April 1967, recalibration of the standard yielded a value of 1.000 013 6 ohms requiring only an

insignificant change of the value assigned to the standard of 1.000 013 47 ohms as shown in Figure 3. The additional point permitted also the narrowing of the two-sigma equivalent control limits to 1.2 microohms as n had now increased to 6 ($n_2 = 6$).



Trend Chart of a Thomas Type Standard Resistor

Figure 3

Having determined 1.) the value to be assigned to the standard after its calibration in April 1967 (1.000 013 47 ohms) until its next calibration and 2.) the range of values in which future calibrated values will be expected to lie, we must now calculate the uncertainty limits about the assigned value in accordance with equation 4:

$$L_U, L_L = \bar{X} \pm \frac{s}{\sqrt{n}} t_{P,\ n-1} \qquad (4)$$

For the first five values, the limits of uncertainty are at 1.000 013 44 ohms ±0.66 microhms, $t_{0.9544,4} = 2.869$. After April 1967, the uncertainty limits about the assigned value are at 1.000 013 47 ohms ±0.51 microohms, $t_{0.9544,5} = 2.649$. These limits include the effects of all experienced sources of variability on the value of the standard and should be considered the uncertainty due to random fluctuations of X.

To arrive at the total uncertainty with which the value of the standard is known to us, we now only have to add the systematic error of the reported value to the uncertainty due to random fluctuations calculated above in equation 4. In our case (Figure 3), the systematic error for the period 1963 through 1969 was reported to be about 0.5 microohms maximum, so that the total uncertainty of the value of 1.000 013 4 ohms, assigned to it in 1965, was about 1.2 microohms (0.66 + 0.5 microohms). This total uncertainty dropped in 1967 to 0.51 + 0.5 = 1.0 microohms.

Although the 0.5 microohms maximum limits on the systematic error may already include some variable component which then would also be included in the uncertainty limits plotted on the control chart, nothing is known about its magnitude, so that prudence dictates that we add the entire systematic error component of NBS to our uncertainty.

As time progresses, trend charts must sometimes be updated to account for changing technology in calibrating the standard, changing measurement techniques, a perhaps slowly changing standard itself, and sometimes changing definitions. Such updating can be accomplished for instance by dropping some of the earliest values or by assigning different weights to the values, with the higher weights being assigned to the latest values and low weights to early values. If, for instance, the standardizing laboratory (e.g., NBS) utilizes calibration techniques yielding significantly narrower limits of uncertainty than before, it may be appropriate to weigh the charted values with a factor inversely proportional to the reported uncertainty limits.

STANDARDS WITH OBSERVED DRIFT

A bit more complicated is the construction of a control chart for standards with an observed drift. If the drift is approximately linear, the best known value of the standard, X, at any time t may be expressed by equation 5.

$$X = \hat{b} + \hat{m}t \qquad (5)$$

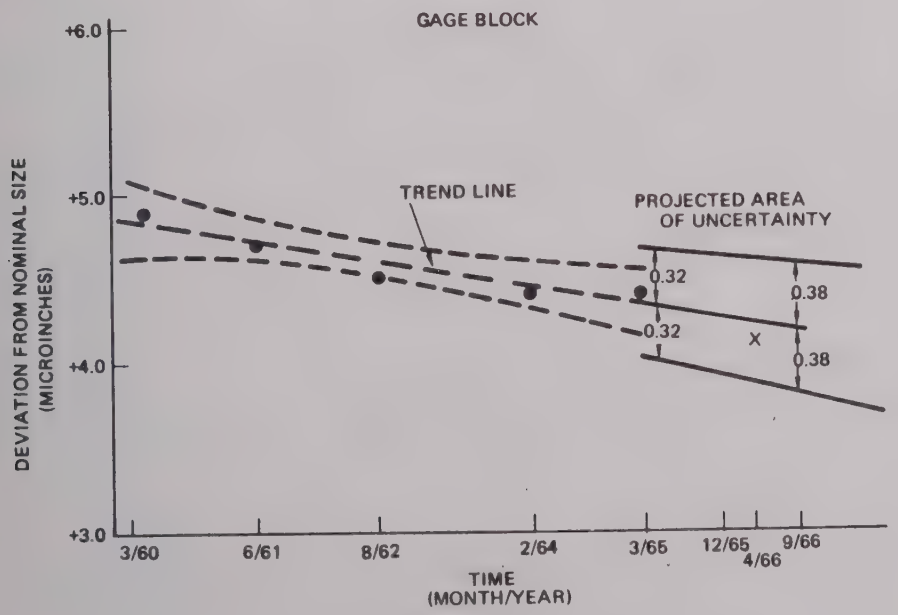
where $\hat{b}$ is the least squares X-intercept and $\hat{m}$ is the least squares slope.

$$\hat{m} = \frac{\sum_{i=1}^{n} X_i t_i - \bar{X} \sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} t_i^2 - \bar{t} \sum_{i=1}^{n} t_i} \qquad \hat{b} = \bar{X} - \hat{m}\bar{t} \qquad (6)$$

The symbols $\bar{t}$ and $\bar{X}$ represent average t and X-coordinates

$$\bar{t} = \frac{1}{n} \sum_{i=1}^{n} t_i \qquad \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \qquad (7)$$

Refer to Figure 4, the trend chart of a gage block suspected to be shrinking gradually after a control chart was plotted and long before any reason for such a shrinkage could be found. The chart was plotted in March 1965 on the basis of five successively reported values. The X indicates a value reported by NBS in April 1966.



Trend Chart of Gage Block

Figure 4

To check whether the calculated least-squares line adequately describes the suspected drift, upper and lower control limits (dashed lines) are drawn (at 95 percent probability limits in Figure 4) at a distance d_k from the trend line for selected past instances t_k calculated from

$$d_k = t_{P,n-2} s_X \left[\frac{1}{n} + \frac{\left(t_k - \bar{t}\right)^2}{\sum_{i=1}^{n} t_i^2 - \bar{t} \sum_{i=1}^{n} t_i} \right]^{1/2} \qquad (8)$$

where $t_{P,\,n-2}$ is the two-sided P-probability value of Student's t for n-2 degrees of freedom and s_X is the estimate of the standard deviation computed from

$$s_X^2 = \frac{1}{n-2} \left[\sum_{i=1}^{n} X_i^2 - \bar{X} \sum_{i=1}^{n} X_i - \frac{\left(\sum_{i=1}^{n} X_i t_i - \bar{t} \sum_{i=1}^{n} X_i \right)^2}{\sum_{i=1}^{n} t_i^2 - \bar{t} \sum_{i=1}^{n} t_i} \right] \qquad (9)$$

These control limits serve mainly to check the adequacy of our selected model. If a past point, which we have established as being valid, falls outside these limits, we should re-examine our model; otherwise we accept it and use it to forecast future values of the standard and their associated limits of uncertainty. (It is assumed here that the uncertainty of the past point due to random variability is negligible with respect to d_k.)

To determine the limits of uncertainty for projected future values of the standard (solid control limits in Figure 4), the distance d_f from the (solid) trend line at any future time t_f is calculated from

$$d_f = t_{P,n-2} s_X \left[1 + \frac{1}{n} + \frac{\left(t_f - \bar{t}\right)^2}{\sum_{i=1}^{n} t_i^2 - \bar{t} \sum_{i=1}^{n} t_i} \right]^{1/2} \qquad (10)$$

The standard is recalibrated whenever the uncertainty about its assigned value, as indicated by the projected limits of uncertainty, becomes as wide as measurement requirements permit, unless other considerations make earlier recalibrations desirable. Whenever a new value is obtained, the trend chart, of course, is updated, trend lines, control limits and projected uncertainty limits are recalculated. Eventually it may also here become desirable to drop some of the oldest points. A decision to drop some old points should be based on identifiable differences between the older points and the later points or the processes by which they were obtained; it should not be made arbitrarily.

Higher order least squares curves with control limits are considerably more difficult to construct. And before embarking on such a project, it is advisable to determine first whether the additional information obtained from a higher order trend chart will be worth the cost. We should bear in mind that the number of degrees of freedom decreases with increasing order of the curve. When the number of points to be fitted is small, the uncertainty of any value on a higher order least squares curve may not be significantly less than that obtained from a linear approximation.

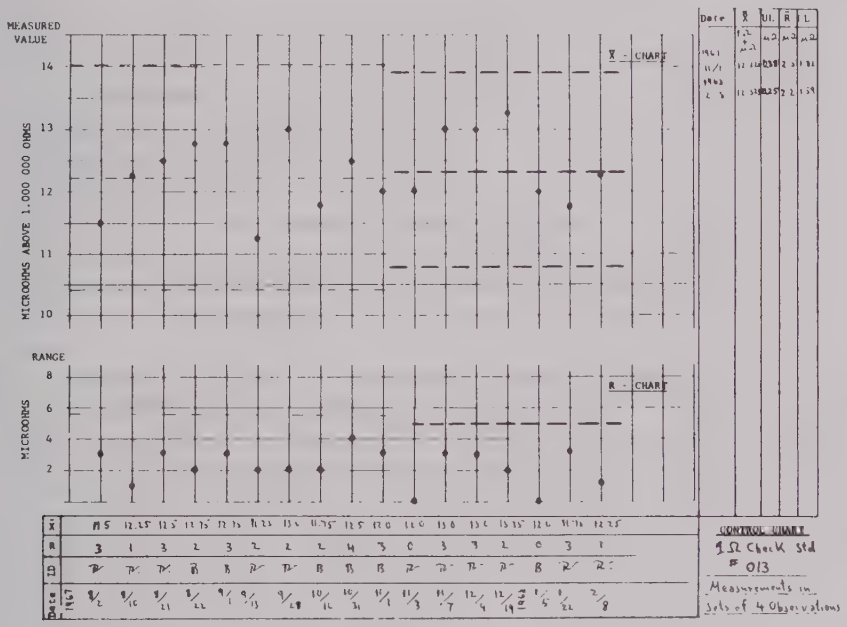
CONTROLLING THE TOTAL MEASUREMENT PROCESS

THE $\bar{X}$ AND R-CHART

One of the most powerful tools to control the entire measurement process are control charts for averages and dispersions (Figure 5).

They can be constructed and maintained whenever one calibration or one measurement value is the average of more than one observation. For example, Figure 5 depicts the measurement process of 1-ohm resistors where each measured value is the average of four observations made on a check standard measured with the same process as the unknown resistors. The observations are made over

a period of 32 to 48 hours. The $\bar{X}$-Chart, the plot of the averages of four observations, is in all respects similar to the control charts shown in Figures 1 and 2, except for the calculations of the control limits. The comments made concerning the simple control charts, including the evaluation criteria, apply here as well.



$\bar{X}$ and R Chart
Control Chart for Averages and Dispersions
Figure 5

But in addition to plotting the measured values, we also plot a measure of the dispersion of the four observations on which each average is based. For instance, the second point on the $\bar{X}$-chart (Figure 5) is plotted as 1.000 012 25 ohms, this value being the average of three observations yielding 1.000 012 ohms and one yielding 1.000 013 ohms.

Beneath the 1.000 012 25 ohm point on the $\bar{X}$-Chart, we plot, therefore, the range of the measurements on the R-Chart, in this case 1 part per million. (The range is the algebraic difference between the largest and the smallest observation from those observations from which the plotted average was calculated.)

Instead of charting the range on an R-Chart, we could also plot the estimates of the standard deviation on an s-Chart. An s-Chart should be used when the number of observations of each point is large, say in excess of ten. But, "The R-Chart is preferred to the (s-Chart) because of its simplicity and versatility; and, unless there are compelling reasons to use the (s-Chart), the R-Chart is the method of choice" (Ref. 4, p. 2-7). "For small samples, the sample range is a reasonably efficient substitute for s as an estimator of the standard deviation of a normal population - not as efficient as s, but easier to calculate..." However, "As the sample size gets larger, the range is not only troublesome to calculate, but is a very inefficient estimator of σ" (Ref. 4, p. 2-7). The range should not be used as an estimator of the standard deviation for samples of more than ten observations (Ref. 5, p. 62).

The center line of the $\bar{X}$ Chart, $\bar{\bar{X}}$, is the (grand) average of the plotted averages. The center line of the R-Chart, $\bar{R}$, is the average of the plotted ranges. With at least ten points available, we enter a preliminary center line and upgrade it to a more permanent status when we have at least 25 points to base our averages on. Upper and lower control limits are drawn at three-sigma from the center line as shown below. (See also Ref. 5.)

We should bear in mind that we are plotting the results of pilot measurements, using a known check standard added to a number of unknown items to be measured, in order to control the quality (accuracy) of the process employed to measure the unknown items. From the verifiable results of measuring the known check standard we draw conclusions about the quality of our measurements of the unknown items.

Although occasionally a point may fall outside the control limits as a matter of pure chance, each time this happens the existence of some abnormality in the measurement is suspected, and the measurement is repeated. If the point of the repeat measurement also falls outside the control limits, the existence of an abnormality is taken as being confirmed. The causes for this abnormality are then determined, removed, and the measurement is repeated. If the points on the $\bar{X}$ and the R-Chart fall within the control limits, the measurement is

considered valid. Since our customers' unknown items are measured by the same process, their values thus determined are also considered valid.

The $\bar{X}$-Chart will reveal the existence of short term trends or cycles, and unusual systematic errors. The R-Chart is predominantly an indicator of the precision of the measuring process, reflecting, among other things, the care of the operator and the control of the environment in which the measurement was performed.

Charts for the range (R-Charts) or the standard deviation can also be used to detect short-term instabilities of the measured items; such a case is illustrated in Ref. 2 (pp. 326-327) for the calibration of standard cells.

Since a control chart is the result of a compilation of a considerable amount of data concerning one measuring process, the average experienced range $\bar{R}$ can be used to determine the standard deviation of the measuring process. "Most experimental scientists have very good knowledge of the variability of their measurements, but hesitate to assume known σ without additional justification. Control charts can be used to provide the justification" (Ref. 4, p. 18-2).

CONSTRUCTING AND EVALUATING THE $\bar{X}$ AND R-CHART

The recent grand average of pilot measurements illustrated in Figure 5 is $\bar{\bar{X}}$ = 1.000 012 3 ohms and the average range is $\bar{R}$ = 2.2 microohms as plotted. Upper and lower control limits on the $\bar{X}$-Chart of the averages are at a distance of $A_2\bar{R}$ = 0.729 x 2.2 = 1.6 microohms from the grand average $\bar{\bar{X}}$. Upper and lower control limits of the R-Chart of ranges are at $D_4\bar{R}$ = 2.282 x 2.2 = 5.0 microohms and $D_3\bar{R}$ = 0 x 2.2 = 0.0 microohms respectively, as plotted. The estimated mean standard deviation of the calibration process, where each measured value is the average of n' observations (in our example n' = 4) and where the observations have an average range $\bar{R}$ is

$$\sigma_{\bar{X}} = \frac{\bar{R}}{d_2 \sqrt{n'}} \qquad (11)$$

and the three-sigma control limits are at

$$3\sigma_{\bar{X}} = \frac{3\,\bar{R}}{d_2\sqrt{n'}} = \frac{3 \times 2.2}{2.059\sqrt{4}} = 1.6 \text{ microohms} = A_2\bar{R}$$

The factors A_2, d_2, D_3, and D_4 are tabulated in Ref. 4 and 5. See also Table 3.

Like all the others, these control charts also must be updated periodically after some new group of points have been entered to permit them to remain responsive to changes in the measurement process as it operates now and to determine whether some more hidden, long-term effects have changed the process. At that time it may also become desirable to drop some of the earlier points.

The check standard whose control chart we are keeping, as in Figures 1, 2, or 5, should be measured or calibrated occasionally by a different process, preferably with higher accuracy. It is then very likely that the value obtained by this process differs from the long-term average, $\bar{\bar{X}}$ in Figure 5 or $\bar{X}$ in Figures 1 and 2, shown as the center line of our control chart. This may mean that our process converges to a value which is somewhat different from a value obtained elsewhere.

The check standard for the process depicted in Figure 5 is calibrated from time to time against the reference standard of the laboratory and is known to be 1.000 012 5 ohms in terms of the laboratory's reference, or 0.2 microohms larger than indicated by the control chart, Figure 6. We must now answer the important question: "Is this difference significant? Does the difference indicate a bias which should be corrected or can such a difference be accounted for by the variability of the process controlled by this check standard?"

We answer this question by noting that the difference is statistically significant at a level α if it is larger than

$$u = Z_P \frac{\sigma_{\bar{X}}}{\sqrt{n}} \qquad (12)$$

where Z_p is the standard normal variable or the ordinate of the normal curve for a two-sided P probability value where P = 1-α. From (11)

$$\sigma_{\bar{X}} = \frac{2.2}{2.059 \quad \sqrt{4}} = 0.534 \text{ microohms}$$

and from (12)

$$u = 1.96 \frac{0.534}{\sqrt{17}} = 0.25 \text{ microohms}$$

Since our difference of 0.2 microohms is less than u, we conclude that the difference is not significant at the 5% level. Otherwise we would have to conclude that our process may have developed a bias which we would have to eliminate before using the process for further measurements.

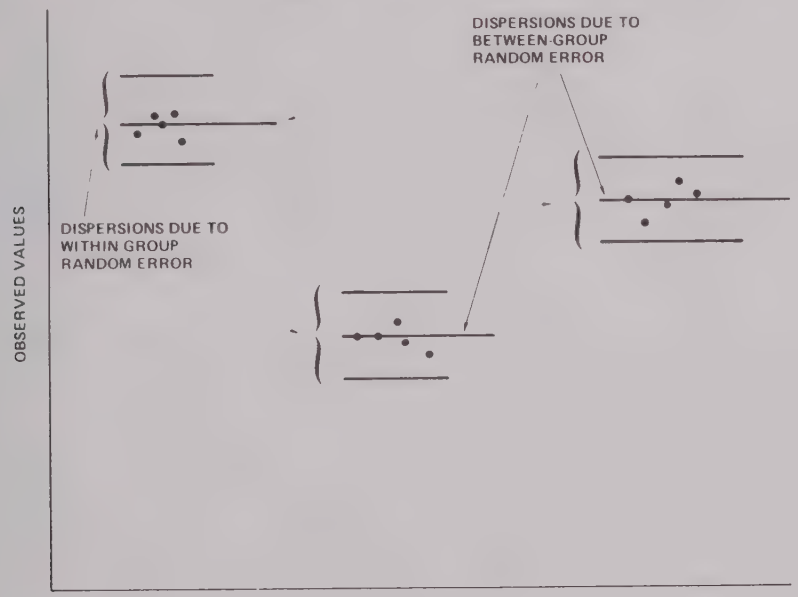
THE "BOTTOM LINE"

The total uncertainty of the process illustrated by the above example is then the sum of the random error and the systematic error. For measurements in sets of four, the random error of our process is 1.6 microohms from Figure 5 and the above calculations.

The systematic error is composed of the total uncertainty of the standard plus any systematic uncertainty contributed by the transfer process. From the above calculation, we have no reason to suspect any systematic uncertainty contribution from the process, so that the total systematic error is the uncertainty of the standard. The standard is the one whose trend chart is shown in Figure 3. We calculated its total uncertainty in 1967 as 1.0 microohms, Hence, we should report the total uncertainty of the measurement as 2.6 microohms, consisting of a process random error of 1.6 microohms, using three standard deviations, plus a systematic error estimated not to exceed 1.0 microohms.

SURPRISES

Metrologists, attempting to obtain an objective measure of the uncertainties of their measurements by the methods illustrated here, frequently meet with surpises caused by "between-group" random error (Figure 6).



Within-Group and Between-Group Random Error
Three Measurements, Each Based on Five Observations

Figure 6

Depending under what conditions observations are repeated, some sources responsible for variations may not be included in the repetitions. Metrologists estimating the random error of their measurements from individual sets of observations would thus underestimate the uncertainty. In fact, the neglect of possible between-group random errors is probably one of the single largest causes of underestimating uncertainties.

$\bar{X}$ and R-Charts will signal the presence of unaccounted for between-group random errors. Since the ranges and the average range $\bar{R}$ depend only on the

within-group random error, a chart constructed as outlined above under "Controlling the Total Measurement Process" will indicate an out-of-control condition in accordance with the criteria described under "The Control Chart". Such an indication is given if several points fall beyond the three-sigma control limits calculated and plotted as A_2R, or too many points fall on one or the other side of one-sigma and two-sigma lines. (These lines should also be plotted on the control chart.)

If that happens, the original control limits as well as one-sigma and two-sigma lines should be discarded and new control limits calculated as shown for trend charts, using equation 4 and the method outlined under "Stable Standards."

Having thus been made aware of the presence of additional error sources between calibrations or measurements, we may investigate the origin of such additional error sources and, after eliminating them, increase the precision of the process. If, on the other hand, the process is deemed to be sufficiently precise, we may consider reducing the number of observations per measurement or calibration. Another recommendable alternative would be to redesign the process so as to include the previously excluded sources of variation. The object here is to make the process and the controls as efficient as possible.

CONCLUSION

Other control charts may be used for different purposes; other evaluation methods have been developed for specific measurement control purposes. For further studies of the subject, the reader is referred to the Bibliography. The control charts discussed here, however, are most valuable in assuring the validity of precision measurements and the quality of the services rendered by metrologists.

Unstable standards cause out-of-control measurement systems, regardless whether the instability is caused by a standard which actually changes or whether it is caused by us when we assign periodically different values to the same thing. Trend charts for reference standards will greatly enhance the stability of our standards by telling us what best values to assign to them; they will also tell us the combined uncertainty of the assigned value which is subject to short-term and longer-term fluctuations, i.e., the total uncertainty except for the systematic error. Control charts for check standards enable us to monitor continuously the quality of our measurement processes, give us an accurate indication of the variability of the measurement processes, indicate promptly when individual measurements had excessive random or systematic errors, and signal rapidly when measurement processes drift out of control.

With control charts we can greatly enhance the quality of our measurements at a small additional effort. And in some cases, control charts can give us assurance of accuracies that cannot be achieved by any other means at any cost.

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TABLE I.

STUDENT'S t FACTOR $t_{P,v}$

Degrees of Freedom v	P: One-Sigma Equivalent 0.6826 *	0.9500	Two-Sigma Equivalent 0.9544 *	0.9900	0.9950	Three-Sigma Equivalent 0.9973 *
1	1.837	12.706	13.968	63.657	127.32	235.80
2	1.321	4.303	4.527	9.925	14.089	19.207
3	1.197	3.182	3.307	5.841	7.453	9.219
4	1.142	2.776	2.869	4.604	5.598	6.620
5	1.111	2.571	2.649	4.032	4.773	5.507
6	1.091	2.447	2.517	3.707	4.317	4.904
7	1.077	2.365	2.429	3.499	4.029	4.530
8	1.067	2.306	2.366	3.355	3.833	4.277
9	1.059	2.262	2.320	3.250	3.690	4.094
10	1.053	2.228	2.284	3.169	3.581	3.957
25	1.020	2.060	2.105	2.787	3.078	3.330
$\infty(z_P)$	1.000	1.960	2.000	2.576	2.807	3.000

*From Reference 3.

TABLE II

EQUATIONS FOR CONTROL CHARTS

Chart	Center Line	Three-Sigma Upper Control Limits	Three-Sigma Lower Control Limits	Remarks
$\bar{X}$-Chart	$\bar{\bar{X}} = \frac{1}{n}\sum_{i=1}^{n} \bar{X}_i$	$\bar{\bar{X}}+A_2\bar{R}$	$\bar{\bar{X}}-A_2\bar{R}$	
		$\bar{\bar{X}}+A'_1\sigma$	$\bar{\bar{X}}-A'_1\sigma$	See Notes 1 and 2
R-Chart	$\bar{R} = \frac{1}{n}\sum_{i=1}^{n} R_i$	$D_4\bar{R}$	$D_3\bar{R}$	$\sigma = \frac{\bar{R}}{d_2}$
s-Chart	$\sigma \doteq \bar{s} = \frac{1}{n}\sum_{i=1}^{n} s_i$	$B_4\sigma$	$B_3\sigma$	$s_i = \sqrt{\frac{1}{n'-1}\left[\sum_{j=1}^{K} X_{ij}^2 - \bar{X}_i \sum X_{ij}\right]}$

Note 1: $A'_1 = \sqrt{\frac{n-1}{n}}\, A_1$ where A_1 is as listed in Ref. 4 and 5

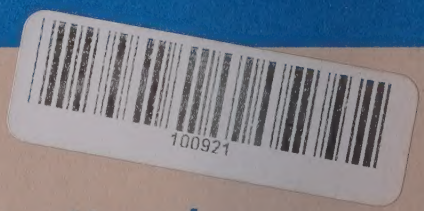
See Note 2.

Note 2: Use only to control consistency of included data; not for comparison against prior data.

TABLE III

FACTORS FOR CONTROL CHARTS

Number of Observations in Sample, n	A'_1	A_2	B_3	B_4	D_3	D_4	d_2
2	2.659	1.880	0	3.267	0	3.267	1.128
3	1.955	1.023	0	2.568	0	2.575	1.693
4	1.628	0.729	0	2.266	0	2.282	2.059
5	1.428	0.577	0	2.089	0	2.115	2.326
6	1.287	0.483	0.030	1.970	0	2.004	2.534
7	1.182	0.419	0.118	1.882	0.076	1.924	2.704
8	1.099	0.373	0.185	1.815	0.136	1.864	2.847
9	1.031	0.337	0.239	1.761	0.184	1.816	2.970
10	0.975	0.308	0.284	1.716	0.223	1.777	3.078



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